Bound Energy Masses of Mesons Containing the Fourth Generation and Iso-singlet Quarks

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Abstract

The fourth Standard Model (SM) family quarks and weak iso-singlet quarks predicted by E_6 GUT are considered. The spin-average of the pseudoscalar $\eta_4(n^1S_0)$ and vector $\psi_4(n^3S_1)$ quarkonium binding masses of the new mesons formed by the fourth Standard Model (SM) family and iso-singlet E_6 with their mixings to ordinary quarks are investigated. Further, the fine and hyperfine mass splittings of the these states are also calculated. We solved the Schrödinger equation with logarithmic and Martin potentials using the Shifted large-N expansion technique. Our results are compared with other models to gauge the reliability of the predictions and point out differences.

I. INTRODUCTION

The toponium quark does not form hadronic states because of its large mass value ($m_t \approx 175$ GeV) and full strength of tbW vertex. On the other hand, there are strong reasons that the fourth Standard Model (SM) family should exist [1,2]. The flavor democracy (Democratic Mass Matrix approach) [2] favors the existence of the nearly degenerate fourth SM family, whereas the fifth SM family is disfavored both by the mass phenomenology and precision tests of the SM [3]. The fourth SM family fermions and also the isosinglet quarks production at $\mu^+\mu^-$ colliders have been investigated [3]. Thus, SM may be treated as an effective theory of fundamental interactions rather than fundamental particles. The multi-hundreds GeV fourth generation up-type quark (u_4), if exist, will be produced at the CERN Large Hadron Collider (LHC) [3,4] via gluon-gluon fusion [4]. Hence, the observation of the fourth SM family quark in ATLAS has been considered in [4,5]. It is expected that the masses of the fourth family

quarks lies between $300 \leq m_{u_4} \leq 700\ GeV$ with preferable value $m_{u_4} = 4gw\eta = 8m_w \cong 640$ GeV [3]. The partial-wave unitarity leads to $m_Q \leq 700GeV = 4m_t$ and in general $m_t \ll m_{u_4} \ll m_5$. The fourth generation up-type quark would predominatly decay via $u_4 \to Wb$, with an expected event topologies are similar to those for $t-$quark pair production. The best channel for observing it will be [5]: $u_4\overline{u}_4 \to WWb\overline{b} \to (l\nu)(jj)(b\overline{b})$, where one W decays leptonically and the other hadronically. The mass resolution is estimated to be 20(40) GeV for $m_{u_4} = 320(640)$ GeV. The pseudoscalar quarkonium state $\eta_4(^1S_0)$ formed by the SM fourth generation quarks is the best candidate among the fourth generation quarkonia to be produced at the LHC and VLHC [6]. On the other hand, new heavy quarks known as weak iso-singlet quarks are also predicted by various extensions of SM and by E_6 GUT [7] which is favored by superstring theory [8]. For down-type quark (d_4), the decay mode is: $d_4 \to Wt$ and the final state contains four W bosons: $d_4\overline{d}_4 \to tW^-\overline{t}W^+ \to bW^+W^-\overline{b}W^-W^+$.

The small inter-family mixings [9] leads to the formation of the fourth family quarkonia. The most promising candidate for LHC is the pseudoscalar quarkonium state, η_4, which will be produced reasonantly via gluon-gluon fusion particularly through decay channel: $\eta_4 \to ZH$ [10].

In spite that the masses of new quarks are larger than m_t, they can form new hadrons because of the small inter-family mixings leads to the formation of the fourth family quarkonia between new heavy and ordinary quarks (u, d, s, c, b). Indeed, according to the parametrization of mass matrices given in [9] mixing between the fourth and third family quarks is predicted to be $|V_{qu_4}| \approx 10^{-3}$. Similar situation is expected for iso-singlet down-type quarks. The condition for forming $(Q\overline{Q})$ quarkonia states with new heavy quarks is [11]

$$m_Q \leq (125\ GeV)\,|V_{Qq}|^{-2/3}\,. \tag{1}$$

where $q = d, s, b$ for $Q = u_4$ and $q = u, c, t$ for $Q = d_4$. Differing from t quark, fourth family quarks will form quarkonia because u_4 and d_4 are almost degenerate and their decays are suppressed by small mixings [2,9,12].

One of the important goals of the present work is to extend the shifted large-N expansion

technique (SLNET) developed for the Schrödinger wave equation [13, 14, 15] and then applied to semirelativistic wave equations [16,17] to reproduce the spectroscopy of the fourth SM generation up-type (u_4) quark and weak iso-singlet down-type (d_4) quark with their small mixings to the ordinary type (u, d, s, c, b) quarks.

Here, we study the present status of the new heavy mesons formed by new quarks in the framework of nonrelativistic potential model and give some predictions for their bound energy masses. In section II, we present the solution of the Schrödinger equation using the flavor-independent logarithmic and Martin potentials for the the self- and non-self conjugate $q_i\overline{q}_i$ and $q_i\overline{q}_j$ mass spectra, respectively. A brief conclusion appear in section III.

II. NEW HEAVY MESONS

The fourth SM family and E$_6$ isosinglet quarks have formed hadron states if their mixing with ordinary known quarks is sufficiently small. In the fourth family quarks, the parametrization given in [9] satisfies condition (1), whereas new hadrons are not formed in the case of paramerization given in [1]. Concerning E$_6$ iso-singlet quarks, we have no similar parametrization (one deals with 6×6 mass matrix) and one can make qualitative estimations. For example, if the lightest isosinglet quark has the mass $m_{d_4} \cong 0.5\ TeV$, new heavy hadrons are formed for $|V_{qd_4}| < 0.09$. In order to calculate the binding masses of new mesons we investigate two potentials: (1) Logarithmic potential of the form [14,16,18]

$$V(r) = -0.6631 + 0.733 \ln(r \times 1GeV), \tag{2}$$

where r is the interquark distance which is singular at $r = 0$. (2) Martin's potential of the form [14,16,19]

$$V(r) = -8.093 + 6.898 r^{0.1}, \tag{3}$$

which behaves in some respects, as its power approaches to zero, as the logarithmic potential. Hence, these potential types are stimulated by the approximate equality of mass splittings [18]

$$M(\Upsilon'(b\overline{b})) - M(\Upsilon(b\overline{b})) \approx M(\psi'(c\overline{c})) - M(\psi(c\overline{c})), \tag{4}$$

which is independent upon reduced mass. These static quarkonium potentials are monotone nondecreasing, and concave functions satisfying the condition [14]

$$V'(r) > 0 \text{ and } V''(r) \leq 0. \tag{5}$$

For two particles system, we shall consider the N$-$dimensional space Schrödinger equation for any spherically symmetric potential $V(r)$. If $\psi(\mathbf{r})$ denotes the Schrödinger's wave function, a separation of variables $\psi(\mathbf{r}) = Y_{\ell,m}(\theta,\phi)u(r)/r^{(N-1)/2}$ gives the following radial equation ($\hbar = c = 1$) [13,14,15,16,17]

$$\left\{-\frac{1}{4\mu}\frac{d^2}{dr^2} + \frac{[\overline{k}-(1-a)][\overline{k}-(3-a)]}{16\mu r^2} + V(r)\right\}u(r) = E_{n,\ell}u(r), \tag{6}$$

where $\mu = \left(m_{q_i}m_{q_j}\right)/(m_{q_i}+m_{q_j})$ is the reduced mass for the two quarkonium composite particles. Here, $E_{n,l}$ denotes the Schrödinger binding energy of meson, and $\overline{k} = N + 2\ell - a$, with a representing a proper shift to be calculated later on and l is the angular quantum number. We follow the shifted $1/N$ or $1/\overline{k}$ expansion method [13,14,15] by defining

$$V(x(r_0)) = \sum_{m=0}^{\infty}\left(\frac{d^mV(r_0)}{dr_0^m}\right)\frac{(r_0x)^m}{m!Q}\overline{k}^{(4-m)/2}, \tag{7}$$

and also the energy eigenvalue expansion [14,15]

$$E_{n,l} = \sum_{m=0}^{\infty}\frac{\overline{k}^{(2-m)}}{Q}E_m, \tag{8}$$

where $x = \overline{k}^{1/2}(r/r_0 - 1)$ with r_0 is an arbitrary point where the Taylor's expansions is being performed about and Q is a scale parameter to be set equal to $\overline{k}^2$ at the end of our calculations. Following the approach presented by Ref.[14], we give the necessary expressions for calculating the binding energies:

$$E_0 = V(r_0) + \frac{Q}{16\mu r_0^2}, \tag{9}$$

$$E_1 = \frac{Q}{r_0^2}\left[\left(n_r + \frac{1}{2}\right)\omega - \frac{(2-a)}{8\mu}\right], \tag{10}$$

$$E_2 = \frac{Q}{r_0^2}\left[\frac{(1-a)(3-a)}{16\mu} + \alpha^{(1)}\right], \tag{11}$$

$$E_3 = \frac{Q}{r_0^2}\alpha^{(2)}, \tag{12}$$

where $\alpha^{(1)}$ and $\alpha^{(2)}$ are two useful expressions given by Imbo *et al* [13] and also the scale parameter Q is defined by the relation

$$Q = 8\mu r_0^3 V'(r_0). \tag{13}$$

Thus, for the $N = 3$ physical space, the Schrödinger binding energy to the third order is [14]

$$E_{n,\ell} = V(r_0) + \frac{1}{2} r_0 V'(r_0) + \frac{1}{r_0^2}\left[\frac{(1-a)(3-a)}{16\mu} + \alpha^{(1)} + \frac{\alpha^{(2)}}{\overline{k}} + O\left(\frac{1}{\overline{k}^2}\right)\right]. \tag{14}$$

where the shifting parameter, a, is defined by

$$a = 2 - (2n_r + 1)\left[3 + \frac{r_0 V''(r_0)}{V'(r_0)}\right]^{1/2}, \tag{15}$$

and the root, r_0, is being determined via

$$1 + 2l + (2n_r + 1)\left[3 + \frac{r_0 V''(r_0)}{V'(r_0)}\right]^{1/2} = \left[8\mu r_0^3 V'(r_0)\right]^{1/2}, \tag{16}$$

where $n_r = n - 1$ is the radial quantum number and n is the principal quantum number. Once r_0 is found via equation (16), then the Schrödinger binding energy of the $q_i\overline{q}_j$ system in (14) becomes relatively simple and straightforward. Hence, the bound state mass of the $q_i\overline{q}_j$ system is written as

$$M(q_i\overline{q}_j)_{nl} = m_{q_i} + m_{q_j} + 2E_{n,l}. \tag{17}$$

The expansion parameter $1/N$ or $1/\overline{k}$ becomes smaller as l becomes larger since the parameter $\overline{k}$ is proportional to n which it appears in the denominator in higher-order correction.

Since systems that we investigate in the present work are often considered as nonrelativistic system, then our treatment is based upon Schrödinger equation with a Hamiltonian

$$H_o = -\frac{\nabla^2}{2\mu} + V(r) + V_{SD}, \tag{18}$$

where V_{SD} is the spin-dependent term taking the simple form (cf. Ref.[14] and the references therein)

$$V_{SD} \longrightarrow V_{SS} = \frac{32\pi\alpha_s}{9m_{q_1}m_{q_2}}\delta^3(\mathbf{r})\mathbf{s}_1.\mathbf{s}_2. \tag{19}$$

The spin dependent potential is simply a spin-spin part and this would enable us to make some preliminary calculations of the energies of the lowest two S-states of the new quarks and their small mixings with the ordinary quarks. The potential parameters in this section are all strictly flavor-independents and are fitted to the low-lying energy levels of $c\overline{c}$ and $b\overline{b}$ systems. The strong coupling constant α_s is fitted to the observed charmonium hyperfine splitting of 117 MeV. The numerical values of α_s for the two potential types have been adjusted in Ref.[14] as

$$\alpha_s^{(L)}(c\overline{c}) \simeq 0.220 \quad \text{and} \quad \alpha_s^{(M)}(c\overline{c}) \simeq 0.251, \tag{20}$$

which are found to be dependent on the potential type. Baldicchi and Prosperi [20] used the standard running QCD coupling expression

$$\alpha_s(\mathbf{Q}) = \frac{4\pi}{\left(11 - \frac{2}{3}n_f\right)\ln\left(\frac{\mathbf{Q}^2}{\Lambda^2}\right)}. \tag{21}$$

with $n_f = 4$ and $\Lambda = 0.2\ GeV$ cut at a maximum value $\alpha_s(0) = 0.35$, to give the right $J/\psi - \eta_c$ splitting of $c\overline{c}$ quarkonium and to treat properly the infrared region. Detail on their numerical works are given in Ref. [20]. Whereas Brambilla and Vairo [21] took in their perturbative analysis $0.26 \leq \alpha_s(\mu = 2GeV) \leq 0.30$.

The potential parameters in (2) and (3) together with the quark masses are obtained from the experimental SAD mass: $\overline{M}(\psi(1S))$ (cf. e.g. Refs.[14,16]). In our calculations we used $m_u = m_d = 0.367\ GeV$, $m_s = 0.561\ GeV$ whereas m_c and m_b masses are given by Ref.[14]. Further, the fourth SM family up-type quark and E_6 isosinglet down-type quark masses are taken as $m_{u_4} = 638.6\ GeV$ [9,22] and $m_{d_4} = 0.5\ TeV$ [22], respectively.

Firstly, we calculate the masses of bound states of $c\overline{c}$ and $b\overline{b}$ in Table 1 for the two potential types. It is seen that our results are in good agreement with the experimental results [23,24]. In Table 2, we present the masses of bound states of quarkonia and mesons formed by the fourth SM family up-type quark and E_6 isosinglet quark, respectively. In Table 3, the predicted mass splittings is compared with Ref.[22]. In Table 4, the hyperfine mass splittings $\psi_4 - \eta_4$ for the first two $S-$states are also given. The best mechanism for the production of heavy mesons formed by u_4 and d_4 quarks is the resonance formation of $3S$ and $4S$ quarkonia at lepton colliders with subsequent decay into corresponding meson-antimeson states.

III. CONCLUSION

We have produced the mesons formed by new heavy quarks. The mass splitting of the logarithmic potentials is found to be independent on the reduced mass and is constant in magnitude for any chosen state for all studied quarkonium families. However, it is seen in Martin's potential that the mass splittings are equal for any chosen state in the fourth family, u_4, quark and isosinglet quark with the same type of ordinary quark mixing. Further, the hyperfine splitting mass is found to be much smaller compared to the large mass of the new mesons. Therefore, the pseudoscalar $\eta_4(n^1S_0)$ and the vector $\psi_4(n^3S_1)$ have nearly same masses. On the other hand, the Cornell potential fails to produce the masses of the $u_4\overline{u}_4$ and $d_4\overline{d}_4$ quarkonium famileis and their hyperfine mass splittings properly because of the small values of the roots r_0 for the lowest states where the Coulombic part of the potential goes to high order fas we take higher order derivatives in the SLNET method.

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TABLES

TABLE I. The pseudoscalar, vector and the spin-averaged masses together with hyperfine splittings for the first two-states of $c\bar{c}$ and $b\bar{b}$ states (in MeV).

States	$c\bar{c}$	$b\bar{b}$	$c\bar{c}$	$b\bar{b}$	$c\bar{c}$	$b\bar{b}$
	Logarithmic:		Martin:		Experiment [23,24]:	
$1S$	3068	9444	3068	9445	3068 ± 2	9448 ± 5
1^3S_1	3097	9460	3097	9461	3097	9460
1^1S_0	2980	9395	2980	9397	2980	$-$
Δ_{1S}	117	65	117	64	117	10017 ± 5
$2S$	3654	10030	3670	10018	3663 ± 5	10023
2^3S_1	3668	10037	3685	10030	3686	
2^1S_0	3614	10008	3625	9994	3622 ± 12[a]	
Δ_{2S}	54	30	60	33	57 ± 8[b]	
$3S$	3976	10352	4021	10351		10350 ± 5
$4S$	4200	10575	4272	10590		10580
$5S$	4370	10746	4469	10777		
$1P$	3505	9881	3505	9861	3525 ± 1	9900 ± 1
$2P$	3878	10254	3907	10243		10261 ± 1

[a]Here we cite Ref.[25].

[b]Here we cite Ref.[26].

TABLE II. Predicted Spin-averaged masses of the bound states formed by the fourth SM family u_4 and isosinglet d_4 quarks (in GeV).

SAD Mass	$u_4\overline{u}_4$	$u_4\overline{u}$	$u_4\overline{s}$	$u_4\overline{c}$	$u_4\overline{b}$	$d_4\overline{d}_4$	$d_4\overline{u}$	$d_4\overline{s}$	$d_4\overline{c}$	$d_4\overline{b}$
Logarithmic:										
$\overline{M}(1S)$	1275.05[a]	639.30	639.34	639.92	642.89	997.94[b]	500.70	500.74	501.32	504.29
$\overline{M}(2S)$	1275.64	639.88	639.92	640.50	643.47	998.53	501.28	501.32	501.90	504.88
$\overline{M}(3S)$	1275.96	640.21	640.24	640.82	643.80	998.85	501.61	501.64	502.22	505.20
$\overline{M}(4S)$	1276.18	640.43	640.47	641.05	644.02	999.07	501.83	501.87	502.45	505.42
$\overline{M}(1P)$	1275.49	639.73	639.77	640.35	643.33	998.38	501.13	501.17	501.75	504.73
$\overline{M}(2P)$	1275.86	640.11	640.15	640.73	643.70	998.75	501.51	501.55	502.13	505.10
$\overline{M}(3P)$	1276.11	640.36	640.39	640.97	643.95	999.00	501.76	501.79	502.37	505.35
Martin:										
$\overline{M}(1S)$	1274.82	638.77	638.80	639.62	642.64	997.69	500.17	500.20	501.02	504.04
$\overline{M}(2S)$	1275.28	639.39	639.42	640.21	643.20	998.15	500.79	500.82	501.61	504.60
$\overline{M}(3S)$	1275.54	639.76	639.78	640.55	643.52	998.42	501.16	501.18	501.95	504.92
$\overline{M}(4S)$	1275.73	640.02	640.03	640.79	643.75	998.61	501.42	501.43	502.19	505.15
$\overline{M}(1P)$	1275.15	639.22	639.25	640.05	643.04	998.03	500.62	500.65	501.45	504.44
$\overline{M}(2P)$	1275.46	639.64	639.66	640.44	643.41	998.33	501.04	501.06	501.84	504.81
$\overline{M}(3P)$	1275.67	639.93	639.94	640.70	643.67	998.55	501.33	501.34	502.11	505.07

[a] Fourth family system scale offset by 2.05 GeV to agree with Ref.[22].

[b] Iso-singlet system scale offset by 1.96 GeV to agree with Ref.[22].

TABLE III. Predicted fine-splittings of S and P levels (in MeV).

Mass splittings	$u_4\overline{u}_4$	[22]	$u_4\overline{u}$	[22]	$u_4\overline{s}$	[22]	$u_4\overline{c}$	$u_4\overline{b}$	$d_4\overline{d}_4$	[22]	$d_4\overline{u}$	[22]	$d_4\overline{s}$	[22]	$d_4\overline{c}$	$d_4\overline{b}$
Logarithmic:																
$2S-1S$	59	59	58	59	58	59	58	58	59	59	58	59	58	59	58	59
$3S-2S$	32	32	33	33	32	32	32	33	32	32	33	33	32	32	32	32
$4S-3S$	22	22	22	22	23	23	23	22	22	22	22	22	23	23	23	22
$2P-1P$	37	–	38	–	38	–	38	37	37	–	38	–	38	–	38	37
$3P-2P$	25	–	25	–	24	–	24	25	25	–	25	–	24	–	24	25
Martin:																
$2S-1S$	46		62		62		59	56	46		62		62		59	56
$3S-2S$	26		37		36		34	32	27		37		36		34	32
$4S-3S$	19		26		25		24	23	20		26		25		24	23
$2P-1P$	31		42		41		39	37	30		42		41		39	37
$3P-2P$	21		29		28		26	26	22		29		28		27	26

TABLE IV. Hyperfine mass splittings (in MeV).

Level	$u_4\overline{u}_4$	$u_4\overline{u}$	$u_4\overline{s}$	$u_4\overline{c}$	$u_4\overline{b}$	$d_4\overline{d}_4$	$d_4\overline{u}$	$d_4\overline{s}$	$d_4\overline{c}$	$d_4\overline{b}$	$c\overline{c}$	$b\overline{b}$
Logarithmic:												
Δ_{1S}	5.69	0.39	0.48	0.78	1.39	6.43	0.49	0.61	0.99	1.78	117(117)[a]	65(86)
Δ_{2S}	2.62	0.18	0.22	0.36	0.64	2.96	0.23	0.28	0.46	0.82	54(57 ± 8)[b]	30(35)
Martin:												
Δ_{1S}	4.09	0.45	0.54	0.89	1.38	4.70	0.57	0.69	1.13	1.76	117	64
Δ_{2S}	2.09	0.23	0.28	0.45	0.71	2.41	0.29	0.35	0.58	0.90	60	33

[a] The quantity in bracket is the experimental or taken from other references.

[b] Here we cite Ref.[26].